
A mathematical model for estimating the life-cycle costs of hydrogen-powered vehicles

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Abstract: This study provides a mathematical model that delivers fundamental data for developing a pricing strategy for fuel cell electric vehicles (FCEVs). A mathematical model that transforms the life-cycle cost of a hydrogen vehicle into the corresponding gasoline vehicle is designed using cost-benefit analysis and life-cycle analysis. The FCEV obtains economic advantages when its life-cycle cost is less than or equal to the life-cycle cost of the corresponding gasoline vehicle. Because there is a trade-off between the FCEV's price and the hydrogen fuel price, the results provide a number of price combinations that can be used for decision-making purposes. Using this model, car makers can develop a number of FCEV pricing scenarios, and policy makers can establish support systems to encourage the market entrance of FCEVs such as a subsidy for purchasing and producing FCEVs and/or hydrogen energy. This study delivers a number of combinations of FCEV-hydrogen fuel pricing combinations, comparing the life-cycle costs of conventional gasoline vehicles and hydrogen fuel cell vehicles.

Keywords: Fuel Cell Electric Vehicle (FCEV), Life-Cycle Cost, Transformation Model, Hydrogen Fuel Price, Price Scenario

1. Introduction

Since the onset of the use of fossil fuels, the global economy has increased dramatically. As a consequence, the increased emission of greenhouse gases has accelerated the problem of global warming. According to the International Energy Agency, the transportation sector accounts for 19 percent of global energy use and 23 percent of carbon dioxide emissions [2]. Further, the price of fossil fuels has become more volatile and unpredictable than ever before.

Advanced countries have initiated a number of efforts to reduce the carbon dioxide emissions of the transportation sector. Governmental bodies have introduced policies to encourage the production and dissemination of green cars, and major car manufacturers have made extensive investments in green technologies. Vehicle experts forecast that fossil-fuel-powered vehicles will gradually be replaced with plug-in hybrid electric vehicles (PHEVs), electric vehicles (EVs), and ultimately fuel cell electric vehicles (FCEVs) that use hydrogen energy.

FCEVs have many advantages over fossil-fuel-powered vehicles. Despite these advantages, significant obstacles, including high production cost and the lack of hydrogen

infrastructure, prevent FCEVs from being widely diffused. Manufacturers of FCEVs must address the former issue in order to introduce the vehicle into the market, while the latter issue, building a geographically dispersed hydrogen infrastructure, is a challenge for the government and energy companies.

To speed up the dissemination of FCEVs, car makers are prepared to introduce the vehicles to the market with a price acceptable to consumers even though huge cost disadvantages (e.g., current investment for research and development, insufficient demand to meet optimal production costs) remain unresolved yet. Challenges for developing an FCEV pricing strategy include understanding the economic benefits for consumers.

To overcome these challenges and construct an accurate pricing strategy, the following questions must be addressed:

- (1). To what extent would consumers accept the FCEV price compared to the price of carbon-based fuel vehicles? No great difference between FCEVs and fossil-fuel-powered vehicles in terms of their function and performance may keep consumers from

perceiving various advantages of a FCEV unless the FCEV is more economically efficient than the alternatives.

- (2). To what extent would consumers accept the hydrogen energy price? With the exception of the purchase price, energy costs account for the largest portion of the total cost of ownership of an FCEV, and therefore have a large impact on the consumers' purchasing decision.

The problem for determining FCEV prices is tightly connected to the prediction of hydrogen fuel prices due to a trade-off between them. In order to answer these primary research questions, this study provides a model that estimates acceptable FCEV and hydrogen prices. These estimates can be used as a source of information concerning pricing and any adjustments necessary to calculate an ex-factory price.

2. Problem Statement

A new product must have either a product differentiation or economic advantages to be accepted by consumers [3] [4]. An FCEV is powered by hydrogen fuel, which pollutes the atmosphere to a much lesser extent than fossil-fuel-powered vehicles; however, the basic functions of FCEVs are quite similar to those of current fossil-fuel-powered vehicles. Therefore, FCEVs will have the least market competition when their economic value for consumers is larger than the value of the alternative.

This study proposes a new mathematical model capable of both delivering an FCEV life-cycle costs parallel to the life-cycle costs of the reference product, and simulating the effects of the FCEV price and hydrogen fuel price on one other. The economic feasibility of FCEVs is analyzed using cost-benefit analysis and life-cycle analysis [5]. The results of work by Simpson suggested that plug-in hybrid vehicles (PIHVs) would be economically feasible only when fossil fuel prices are very high and PIHV battery prices are very low [1]. Researchers found that both hybrid and the electric vehicles would be economically feasible (compared to conventional and hydrogen-powered vehicles) if electricity could be produced efficiently enough [6]. These studies suggested that the economic feasibility of a vehicle ultimately depends on the price of energy.

Given the previous findings, this study first develops a simultaneous estimation of hydrogen energy price and FCEV price. In other words, the cost of FCEV's initial investment and the price of hydrogen fuel are the two variables to be calculated. The steps of the analysis are as follows:

- (1). Select a fossil fuel-powered reference vehicle that has the functions similar to the FCEV.
- (2). For each vehicle, calculate the total life-cycle costs.
- (3). Compare the two resulting costs.
- (4). Define the economically feasible FCEV price in consideration of the hydrogen price. The feasible

price is the one at which the FCEV's total life-cycle cost is either the same as or less than the life-cycle cost of reference vehicle.

3. Mathematical Formulation

3.1. Life-Cycle Costs of a Vehicle

The total cost of a given vehicle (k) is decided by the initial investment cost (purchase price, I_k), the annual operating cost (O_k), and the residual value (R_k) of the vehicle. The total cost (C_{Tk}) of a vehicle over its life cycle is calculated in the following equation:

$$C_{Tk} = I_k + \beta_1 O_k - \beta_2 R_k \quad (1)$$

In the above equation, β_1 is the discount rate, which is the annual interest divided by the annual operating cost for n years (t_n) including the interest at a given time (t_0). β_2 is the discount rate, which is the annual interest divided by the residual value of the vehicle after n years (t_n) including the interest at a given time (t_0). The following equations provide the discount rates, given the real discount rate (i):

$$\beta_1 = 1/i (1 - 1/(1+i)^n) \quad (2)$$

$$\beta_2 = 1/(1+i)^n \quad (3)$$

The annual operating cost (O_k) can be expressed with annual fuel cost (F_k), annual tax (T_k), annual maintenance cost (M_k), and annual insurance cost (A_k) incurred during the vehicle's life time.

$$O_k = F_k + T_k + M_k + A_k \quad (4)$$

The annual energy cost (F_k) is defined by considering driving distance (D) per year, fuel price (f_k), and fuel consumption rate (E_k) as follows:

$$F_k = D f_k E_k \quad (5)$$

3.2. Economic Feasibility of FCEVs'

Consequently, FCEVs provide consumers with better economic advantages than conventional gasoline vehicles when their total cost (C_{hydrogen}) is smaller than or equal to the total cost of conventional gasoline vehicles (C_{gasoline}), as shown in the following equations:

$$C_{\text{hydrogen}} \leq C_{\text{gasoline}} \quad (6)$$

$$I_{\text{hydrogen}} + \beta_1 O_{\text{hydrogen}} - \beta_2 R_{\text{hydrogen}} \leq I_{\text{gasoline}} + \beta_1 O_{\text{gasoline}} - \beta_2 R_{\text{gasoline}} \quad (7)$$

4. Case Study

In order to illustrate the capabilities of the proposed model, the results of a hypothetical case study (the hydrogen vehicle's initial investment cost and the hydrogen fuel price in South Korea) are presented. A conventional gasoline vehicle is used as the reference product.

4.1. Variables and Assumptions

The economic feasibility of an FCEV is dependent on the price of hydrogen fuel as shown above in equations (1), (4), and (5). Because hydrogen fuel energy for transportation has not yet been offered commercially, its price as well as the initial investment cost of the FCEV, is defined as a variable.

In order to apply the proposed mathematical model easily, the assumptions are made as follows (summarized in Table 1):

- (1) The gasoline vehicle and the FCEV in this case study meet the same performance standard.
- (2) The initial investment cost of the FCEV consists of the purchasing price (90 percent) and the vehicle registration tax (TRA, 10 percent).
- (3) The FCEV is purchased in 2012 (t_0) and used for 10 years (10 year life-time).
- (4) The residual value of the FCEV in 10 years is eight percent of the purchasing price as defined by the car tax rules in South Korea
- (5) The replacement cost of the hydrogen fuel cell is not considered in this analysis.
- (6) The average driving distance is 15,000 km per year.
- (7) The real discount rate (i) is 2.85 percent.
 $i = (1+i')/(1+I) - 1$
 i : Average price increase of 3.55 percent from 1993-2009 in South Korea
 i' : Interest rate of 6.50 % in the year 1993-2009 in South Korea
- (8) The discount rate β_1 is 8.60 according to equation (2).
- (9) The discount rate β_2 is 0.755 according to equation (3).
- (10) The gasoline price remains US\$ 1.80 per liter for 10 years. The impact of gasoline price changes on the total cost will be illustrated with a sensitivity analysis in section 4.3.
- (11) The price of hydrogen fuel remains unchanged for 10 years.
- (12) The fuel consumption ratios of the gasoline vehicle and the FCEV are 11.7km/l and 117km/kg, respectively.
- (13) Annual tax, annual maintenance cost, and annual insurance cost for the FCEV are as the same as for the fossil-fuel-powered vehicle.

Table 1. Summary of the assumptions.

Description	Value	Unit
n Duration	10	year
i Real discount rate	2.85	%
β_1 Discount rate 1	8.60	
β_2 Discount rate 2	0.755	
r_s Residual value based on the vehicle price	8	%
D Driving distance per year	15,000	km/year
f_{gasoline} Gasoline price	1.80	US\$/l
E_1 Fuel consumption ratio of gasoline vehicle	11.7	km/l
E_2 Fuel consumption ratio of FCEV	117	km/kg

4.2. Results of the Analysis

4.2.1. Life-Cycle Cost for Gasoline Vehicles

Table 2 presents the initial investment cost, the annual operation cost, and the residual value for the gasoline vehicle according to equations (1), (4), and (5). The average annual taxes and insurance costs for the corresponding vehicles in 2009 are used to calculate the annual operating costs; to simplify the analysis, variations in car insurance cost incurred due to different conditions such as age, driving experience, and the particular models of vehicle insured are not considered. Annual maintenance cost is estimated using the standard maintenance service price and parts exchange cycle provided by the Korea Vehicle Maintenance Association.

Table 2. Initial investment cost, annual operating cost, and residual value for a gasoline vehicle.

Classification	Value (US\$)	Sum (US\$)
Initial investment cost (I_{gasoline})	Vehicle's purchasing price	18,715
	Vehicle registration tax	2,079
	Annual fuel cost (F_{gasoline})	2,300
Annual operation cost (O_{gasoline})	Annual car tax (T_{gasoline})	461
	Annual maintenance cost (M_{gasoline})	626
	Annual insurance cost (A_{gasoline})	746
Residual value (R_{gasoline})	1,496	1,496

The life-cycle cost of the gasoline vehicle is estimated using equation (1), initial investment cost, net present value of the residual value, and net present value of the annual operation cost, as shown in Table 3.

Table 3. Life cycle cost of the gasoline vehicle for 10 years.

Classification	Value (US\$)	Sum (US\$)
Life cycle cost (C_{gasoline})	Initial investment cost (I_{gasoline})	20,794
	Net present value of the annual operation cost ($\beta_1 O_{\text{gasoline}}$)	35,544
	Net present value of the residual value ($\beta_2 R_{\text{gasoline}}$)	1,129

4.2.2. Corresponding FCEV Life-Cycle Cost

In order to estimate the FCEV's life-cycle cost, this study focuses on the FCEV initial investment cost (x_{hydrogen}) and hydrogen fuel price (y_{hydrogen}) as variables to be estimated. Table 4 presents the estimates:

Table 4. Initial investment cost, annual operating cost, and residual value for a FCEV.

Initial investment cost (I_{hydrogen})		Value (US\$)	Description
		x_{hydrogen}	
Annual operating cost (O_{hydrogen})	Annual fuel cost (F_{hydrogen})	$128y_{\text{hydrogen}}$	(15,000 + 117k m/kg) x hydrogen unit price
	Annual car tax (T_{hydrogen})	461	Equal to the gasoline vehicle
	Annual maintenance cost (M_{hydrogen})	626	
	Annual insurance cost (A_{hydrogen})	746	
Residual value (R_{hydrogen})		$0.9 \times 0.08 \times x_{\text{hydrogen}}$	

The life-cycle cost for the FCEV is estimated using equation (1). The detailed formulation is as follows:

$$\begin{aligned}
 C_{\text{hydrogen}} &= I_{\text{hydrogen}} + \beta_1 O_{\text{hydrogen}} - \beta_2 R_{\text{hydrogen}} \\
 &= x_{\text{hydrogen}} + \beta_1 (F_{\text{hydrogen}} + T_{\text{hydrogen}} + M_{\text{hydrogen}} + A_{\text{hydrogen}}) \\
 &\quad - \beta_2 (0.9r_s x_{\text{hydrogen}}) \\
 &= x_{\text{hydrogen}} + 8.60(128y_{\text{hydrogen}} + 461 + 626 + 746) \\
 &\quad - 0.755(0.072x_{\text{hydrogen}}) \\
 &= 0.9456x_{\text{hydrogen}} + 1100.8y_{\text{hydrogen}} + 15,764 \quad (8)
 \end{aligned}$$

Table 5 presents the life-cycle cost of the FCEV according to equation (8). A set of initial investment costs from US\$25,000–65,000 per FCEV and a hydrogen fuel price between US\$3.00–10.00 per kg are used. The FCEV has the same or better economic advantages when its life-cycle cost is less than or the same as the corresponding gasoline vehicle's life-cycle cost, estimated at US\$57,467 (see the measures in bold font in Table 5).

Table 5. Life cycle cost of a FCEV.

x_{hydrogen} \ y_{hydrogen}	25,000	35,000	45,000	55,000	65,000
3.00	42,706	52,162	61,618	71,074	80,530
4.00	43,807	53,263	62,719	72,175	81,631
5.00	44,908	54,364	63,820	73,276	82,732
6.00	46,009	55,465	64,921	74,377	83,833
7.00	47,110	56,566	66,022	75,478	84,934
8.00	48,210	57,666	67,122	76,578	86,034
9.00	49,311	58,767	68,223	77,679	87,135
10.00	50,412	59,868	69,324	78,780	88,236

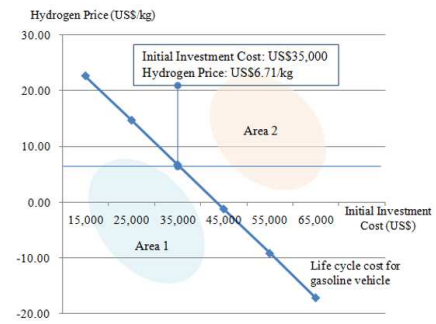
*Hydrogen price (y_{hydrogen}) US\$/kg, Initial investment cost (x_{hydrogen}) US\$.

Given the set of the initial investment costs and the hydrogen fuel price, the economic advantage can also be expressed as the following equation:

$$C_{\text{hydrogen}} = 0.9456x_{\text{hydrogen}} + 1100.8y_{\text{hydrogen}} + 15,764 \leq 57,467$$

$$y_{\text{hydrogen}} \leq -0.000859x_{\text{hydrogen}} + 37.9 \quad (9)$$

Figure 1 illustrates the range of results of equation (9), which transforms the FCEV life-cycle cost into the gasoline vehicle's life-cycle cost. If the x and y coordinate lay directly on the line in the graph, the economic feasibility of both vehicles are the same. For example, the combination of the initial investment cost of US\$35,000 (x) and the hydrogen price of US\$6.71 per kg (y) fits this criterion. In area 1 (below the line), all the combinations of FCEV's initial investment costs and hydrogen fuel prices are more economically advantageous than the corresponding cost of a gasoline vehicle. In contrast, area 3 (above the line), FCEVs are economically disadvantageous.

**Figure 1.** Economically feasible area of the FCEV compared to the gasoline vehicle.

4.3. Sensitivity Analysis

This study uses an estimated gasoline price based on past prices in South Korea. The biggest factor in the price of gasoline is the growth in global demand, and this demand has indeed been increasing recently. In order to explore the effects of variation in gasoline prices on the life-cycle cost, a sensitivity analysis is conducted.

Table 6 presents the annual gasoline fuel cost, the annual operation cost, and the total life-cycle cost based on a set of gasoline prices applying equations (5), (4), and (1), respectively. If the gasoline price of US\$1.80 per liter increases approximately 50 percent to US\$2.80 per liter, the gasoline vehicle's life-cycle cost increases approximately 20 percent, from US\$60,431.92 to US\$72,355.00. Therefore, higher gasoline prices result in greater economic advantages of the FCEV.

Table 6. Results of sensitivity analysis for the gasoline vehicle's life cycle cost (US\$).

gasoline price per liter	Annual fuel cost	annual operation cost	life cycle cost
1.80	2,307.69	4,140.69	60,431.92
2.30	2,948.72	4,781.72	66,393.46
2.80	3,589.74	5,422.74	72,355.00

5. Discussion and Implications

The proposed mathematical model provides a combination of the hydrogen fuel price and the initial investment cost that lead to an FCEV with the same level of economic advantage as a corresponding gasoline vehicle. Although FCEVs have not yet entered the market, and car makers have not been able to bring the initial price down to the level of gasoline vehicles, a pricing strategy must be devised in preparation for the introduction of FCEVs to the market in near future. Because the FCEV price accounts for 90 percent of the initial investment cost according to the proposed model, car makers can use the proposed model to estimate an appropriate price level in reference to the price of hydrogen fuel. For instance, if hydrogen fuel is priced at US\$6.71 per kg, the price of the FCEV should be set at US\$3,150 (90 percent of the initial investment cost at US\$3,500), as shown in Figure 1. In other words, if the FCEV price is set US\$3,150, hydrogen fuel should be offered at US\$6.71 per kg.

In order to estimate the price of FCEV and hydrogen fuel, several assumptions such as a particular real discount rate, residual value based on the vehicle price, gasoline price, and fuel consumption ratio must be made according to the conditions specific to each nation. Because these values differ from country to country, the detailed results of the life-cycle cost for gasoline vehicles might also vary.

At this time, car makers are not capable of achieving the proposed FCEV prices and hydrogen fuel prices in area 1. In order to expand the use of environment friendly cars, policy makers, car makers, and energy companies must enhance the attractiveness of FCEVs. To achieve an appropriate consumer price level, policy makers need to develop nationwide support systems such as tax advantages (e.g., a reduction or waiver of the car registration tax and annual car tax for those purchasing FCEVs) and financial funding to car makers. Further, governmental bodies must invest for the construction of hydrogen infrastructure including the production, storage and transportation of hydrogen fuel.

Because there is a trade-off between the FCEV price and the price of hydrogen fuel, car makers and energy companies are required to establish the optimal Pareto solutions that can be used for decision-making processes. In the absence of clear guidance as to how to provide hydrogen fuel, car makers cannot realistically offer FCEVs in the market, and without demand for FCEVs energy companies cannot construct the hydrogen infrastructure due to the enormous investment cost and risks. Therefore, car makers and energy companies must share these risks.

6. Conclusion

Hydrogen technology is the preferred alternative to

replace the existing carbon-based energy system. Fuel cell electric vehicles are often mentioned as the “next big thing” to address current environmental problems. This study provides fundamental data for an FCEV pricing strategy, and delivers various combinations of FCEV prices and hydrogen fuel prices to assist in comparison of the life-cycle costs of conventional gasoline vehicles and hydrogen fuel cell vehicles. A mathematical model that transforms the life-cycle cost of a hydrogen vehicle into the corresponding cost of a gasoline vehicle is designed using cost-benefits analysis and life-cycle analysis. The FCEV becomes economically advantageous when its life-cycle cost is smaller than or equal to, for example, US\$ 57,467 which is the life-cycle cost of the corresponding gasoline vehicle. Because there is a trade-off between FCEV price and the price of hydrogen fuel, the results provide a number of price combinations that can be used for decision-making purposes. This study also provides various managerial implications necessary to stimulate hydrogen technology for policy makers, car makers, and energy companies.

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